

## Ch 10 Exponential and Logarithmic Functions

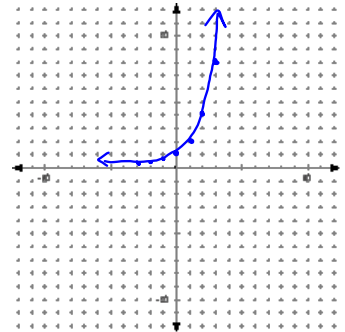
## 10-1 Exponential Functions

Exponential function--function with a variable in the exponent

ex

$$y = 2^x$$

x	y
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4

Form  
 $y = a b^x$ 

$$y = 2^x$$

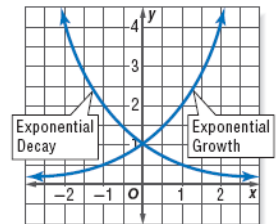
$$a = 1$$

$$b = 2$$

 $a \neq 0$  $b > 0$  $b \neq 1$ 

graph on calculator

Turn plot off  
Clear y=  
Zoom: 6

There are 2 types of exponential functions:  
exponential growth and exponential decay.

## Key Concept

## Exponential Growth and Decay

- If  $a > 0$  and  $b > 1$ , the function  $y = ab^x$  represents exponential growth.
- If  $a > 0$  and  $0 < b < 1$ , the function  $y = ab^x$  represents exponential decay.

## Characteristics (p524)

- The function is continuous and one-to-one.
- The domain is the set of all real numbers.
- The x-axis is an asymptote of the graph.
- The range is the set of all positive numbers if  $a > 0$  and all negative numbers if  $a < 0$ .
- The graph contains the point  $(0, a)$ . That is, the y-intercept is  $a$ .
- The graphs of  $y = ab^x$  and  $y = a\left(\frac{1}{b}\right)^x$  are reflections across the y-axis.

Write an exponential function whose graph passes through the given points.

 $(0, 5)$   $(2, 45)$  $a = 5$ 

$$y = a \cdot b^x$$

$$y = 5 b^x$$

$$45 = 5 b^2$$

$$9 = b^2$$

$$\pm 3 = b$$

$$y = 5(3)^x$$

Write an exponential function whose graph passes through the given points.

(0, 4) (3,  $\frac{1}{2}$ )

$$\begin{aligned} y &= a \cdot b^x \\ y &= 4 \cdot b^x \\ \frac{1}{2} &= 4 \cdot b^3 \\ \frac{1}{8} &= b^3 \\ \frac{1}{2} &= b \end{aligned}$$

$$y = 4\left(\frac{1}{2}\right)^x$$

Solving exponential equations.

ex

$$8^x = \frac{1}{4}$$

$$(2^3)^x = 2^{-2}$$

$$\begin{aligned} 3x &= -2 \\ x &= -\frac{2}{3} \end{aligned}$$

$$\frac{1}{4} = \frac{1}{2^2} = 2^{-2}$$

Check

$$\begin{aligned} 8^{-\frac{2}{3}} &= \frac{1}{4} \\ 2^{-2} &= \frac{1}{4} \\ \frac{1}{4} &= \frac{1}{4} \checkmark \end{aligned}$$

ex

$$5^{4-t} = 25^{t-1}$$

$$5^{4-t} = (5^2)^{t-1}$$

Same Base

$$4-t = 2t-2$$

$$6 = 3t$$

$$2 = t \checkmark$$

ex

$$2^{\frac{5}{3}}$$

$$\sqrt[3]{32} = 2^x$$

$$\sqrt[3]{2^5} = 2^x$$

$$2^{\frac{5}{3}} = 2^x$$

$$\frac{5}{3} = x \checkmark$$

ex

$$9^{\frac{3}{2}}$$

$$9^{x-3} = 27$$

$$(3^2)^{x-3} = 3^3$$

$$2x-6 = 3$$

$$x = 4.5 \checkmark$$

Do:

$$1. 4^{x-2} = 64^x$$

$$4^{x-2} = 4^{3x}$$

$$x-2 = 3x \checkmark$$

$$-1 = x \checkmark$$

$$2. 4^{2x+5} = 16^{x+1}$$

$\emptyset$

## Exponential Inequalities

ex

$$3^x < \frac{1}{27}$$

$$3^x < 3^{-3}$$

$$x < -3$$

ex

$$5^x > \sqrt{125}$$

$$5^x > \sqrt{5^3}$$

$$5^x > 5^{\frac{3}{2}}$$

$$x > \frac{3}{2}$$

ex

$$25^{2x} < 5^{(x+6)}$$

$$(5^2)^{2x} < 5^{(x+6)}$$

$$4x < x + 6$$

$$3x < 6$$

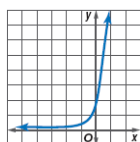
$$x < 2$$

- OPEN ENDED** Give an example of a value of  $b$  for which  $y = b^x$  represents exponential decay.
- Identify each function as *linear*, *quadratic*, or *exponential*.
  - $y = 3x^2$
  - $y = 4(3)^x$
  - $y = 2x + 4$
  - $y = 4(0.2)^x + 1$

Match each function with its graph.

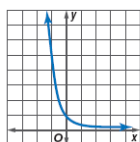
3.  $y = 5^x$

a.



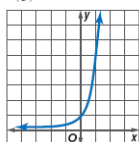
4.  $y = 2(5)^x$

b.



5.  $y = \left(\frac{1}{5}\right)^x$

c.



HW

p528-529

21, 25, 27-29, 33, 35, 39-53odd